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MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

34. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

A particle is placed within a thin cylindrical shell without ends. Find the resultant attraction, the cylinder being composed of matter attracting according to the laws of nature.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Arkansas-Texas.

Let $r=2a\sin\theta$ be the equation to the cylinder, so that the origin is in the surface of the cylinder at one end, then $y=r\sin\theta=2a\sin^2\theta$, $z=r\cos\theta=2a\sin\theta\cos\theta$, l=length of cylinder, (x, y, z) coordinates of any point in the shell, ρ=density, k=thickness of shell. It is always possible to take the axes of coordinates so that the particle will lie in the plane of the axis of y; let (m, n, o) be the coordinates of the particle, mass unity.

$$ds=2adxd\theta,\ p=\sqrt{(x-m)^2+(y-n)^2+z^2}=\sqrt{(x-m)^2+n^2-4a(n-a)\sin^2\theta},\ n>a.$$

This will give attraction for all possible positions of the particle. For $n<a$,

$$p=\sqrt{(x-m)^2+(2a-n)^2-4a(a-n)\sin^2(\tfrac{1}{2}\pi-\theta)},$$

and the solution would be the same as for $n>a$.

$$\text{Let } \frac{4a(n-a)}{m^2+n^2}=b^2,\quad \frac{4a(n-a)}{(l-m)^2+n^2}=c^2,\quad \frac{4a(n-a)}{n^2}=d.$$

Resolving the attractions parallel to the axes, we easily get

$$X=2a\rho k\int_0^\pi\int_0^l\frac{(x-m)d\theta dx}{\{(x-m)^2+n^2-4a(n-a)\sin^2\theta\}^{\frac{3}{2}}}$$

$$=2a\rho k\int_0^\pi\left\{\frac{1}{\sqrt{m^2+n^2-4a(n-a)\sin^2\theta}}-\frac{1}{\sqrt{(l-m)^2+n^2-4a(n-a)\sin^2\theta}}\right\}d\theta$$

$$=\frac{2a\rho k}{\sqrt{m^2+n^2}}E_0^\pi(b,\theta)-\frac{2a\rho k}{\sqrt{(l-m)^2+n^2}}E_0^\pi(c,\theta).$$

$$Y=2a\rho k\int_0^\pi\int_0^l\frac{(2a\sin^2\theta-n)d\theta dx}{\{(x-m)^2+n^2-4a(n-a)\sin^2\theta\}^{\frac{3}{2}}},$$

$$=2a\rho k\int_0^\pi\left\{\frac{l-m}{\sqrt{(l-m)^2+n^2-4a(n-a)\sin^2\theta}}\right.$$

$$\left.+\frac{m}{\sqrt{m^2+n^2-4a(n-a)\sin^2\theta}}\right\}\frac{(2a\sin^2\theta-n)d\theta}{n^2-4a(n-a)\sin^2\theta}$$

$$=\frac{a\rho kn(2a-n)}{n-a}\int_0^\pi\left\{\frac{l-m}{\sqrt{(l-m)^2+n^2-4a(n-a)\sin^2\theta}}\right.$$

$$\left.+\frac{m}{\sqrt{m^2+n^2-4a(n-a)\sin^2\theta}}\right\}\frac{d\theta}{n^2-4a(n-a)\sin^2\theta}.$$

$$-\frac{a\rho k}{n-a}\int_0^\pi\left\{\frac{l-m}{\sqrt{(l-m)^2+n^2-4a(n-a)\sin^2\theta}}+\frac{m}{\sqrt{m^2+n^2-4a(n-a)\sin^2\theta}}\right\}d\theta,$$

$$\therefore\ Y=\frac{a\rho kn(2a-n)}{n^2(n-a)}\left[\frac{l-m}{\sqrt{(l-m)^2+n^2}}\Pi_0^\pi(-d,\,c,\,\theta)\right.$$

$$\left.+\frac{m}{\sqrt{m^2+n^2}}\Pi_0^\pi(-d,\,b,\,\theta)\right]$$

$$-\frac{a\rho k(l-m)}{(n-a)\sqrt{(l-m)^2+n^2}}E_0^\pi(c,\,\theta)-\frac{a\rho km}{(n-a)\sqrt{m^2+n^2}}E_0^\pi(b,\,\theta).$$

$$Z=2a\rho k\int_0^l\int_0^\pi\frac{2a\sin\theta\cos\theta dxd\theta}{\{(x-m)^2+n^2-4a(n-a)\sin^2\theta\}^{\frac{3}{2}}}=0.$$

F=resultant attraction $=\sqrt{X^2+Y^2+Z^2}$.

When $n=a$, the particle is on the axis of the cylinder, then

$$F=X=2\pi a\rho k\left\{\frac{1}{\sqrt{m^2+a^2}}-\frac{1}{\sqrt{(l-m)^2+a^2}}\right\}.$$

When $m=\frac{1}{2}l$, the particle is at the center of the cylinder, and $F=0$.

When $m=l$, $n=a$, $F=2\pi a\rho k\left\{\frac{1}{\sqrt{l^2+a^2}}-\frac{1}{a}\right\}$.

When $m=0$, $n=a$, $F=-2\pi a\rho k\left\{\frac{1}{\sqrt{l^2+a^2}}-\frac{1}{a}\right\}$.

When $n=2a$ the particle is on the surface of the cylinder,

$$\text{then } b^2=\frac{4a^2}{m^2+4a^2},\quad c^2=\frac{4a^2}{(l-m)^2+4a^2},\quad d=1.$$

$\therefore$ The elliptic function of the third order in Y disappears.

PROBLEMS.

42. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Find the time of vibration of a particle *slightly* displaced from the center of a solid cylinder in direction of the axis, the matter of the cylinder attracting according to the laws of nature.

43. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

Two weights P and Q rest on the concave side of a parabola whose axis is horizontal, and are connected by a string, length l, which passes over a smooth peg at the focus, F. [*Bowser's Analytic Mechanics, page 54.*]

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

42. Proposed by W. B. ESCOTT, 6123 Ellis Avenue, Chicago, Illinois.

In a parallelogram, sides a and b, diagonals c and d, $2a^2+2b^2=c^2+d^2$. Find all the parallelograms, not rectangles, whose sides and diagonals are rational.

Examples:	a	b	c	d
	4	7	9	7
	16	7	21	13
	8	9	13	11
	8	11	17	9